Reconstructing Equations of the Rossler's System from the y-variable

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Abstract. A simple way to determine the parameters of Rössler's system based on a suitable output (y-variable) is presented in this paper. The fact that the nonlinear system is observable and algebraically identifiable, with respect to the selected output, allows us to propose, in a first stage, a high-gain observer to estimate the output's time derivatives. And then, based on these facts two suitable schemes to recover the parameters are presented.

Keywords. Chaos; Parameter identification , States reconstruction and Genetic Algorithms.

1 Introduction

Reconstruction of chaotic attractors, from one or more available variables, is an interesting and challenging problem, which has attracted the attention of many researchers because of its theoretical and practical importance (see [Baker *et al.*, 2001], [Broomhead & King, 1986], [Gibson *et al.*, 1992], [Poznyak *et al.*, 1999], [Lainscsek & Gorodnitsky, 1996] and [Crutchfield & McNamara, 1987]). The objective is to find an inverse of mathematical or empirical models; that is, extract, from a partial knowledge of these models, the underlying dynamics ([Parlitz, 1995]). This problem is important: first, experimentally, in general it is not possible to measure or observe the complete state of a given system, or in some cases only a few physical parameters are available (see [Stojanovski *et al.*, 1996]); second, it allows us to verify the accuracy of some empirically derived models (see [Lainscsek & Gorodnitsky, 1996]); and third, to verify how secure a communication system is when the encoding system is chaotic (see [Stojanovski *et al.*, 1996], [Parlitz *et al.*,1994] and [Kocarev *et al.*, 1992]). Roughly speaking, the problem has been solved in two ways. The first approach has been, so far, dominated by the delay embedding methodology founded on the delay reconstruction of states known from non-linear time series analysis (for general background on this fascinating area, the reader is referred to the easily readable books [Alligood *et al.*, 1997] and [Hand & Berthold, 2002]). Also, we recommend reading the papers ([Sauer *et al.*,1991], [Takens, 1981], [Tokuda *et al.*, 2002], [Stojanovski *et al.*, 1996], [Parlitz *et al.*, 1994], [Broomhead & King, 1986] and [Hand & Berthold, 2002]). The other approach is based on control theoretical ideas, such as inverse system design and system identification, as devices to recover parameters and unknown or difficult-to-measure states ([Feldmann *et al.*, 1996], [Poznyak *et al.*, 1999], [Poznyak *et al.*, 1998] , [Cheng & Dong, 1995] and [Suárez *et al.*, 2003]).

In this paper, we deal with the problem of recovering Rossler's parameters by means of the knowledge of an available output (*which is the* $y-$ *variable of the well-known Rossler's model*). The on-line identification approach is based on the algebraic properties of observability and identifiability that Rossler's model satisfies. Those properties allow us to find a differential parameterization of the output and a finite number of its time derivatives. This parameterization is used in two identification approaches: in the first we use a traditionally least-square method which is solved by an *ad-hoc* genetic algorithm (GA). In the second approach, we assume that a specific parameter is given, then an algebraic on-line parametric identification can be formulated. Although the two approaches require the non-available output's time derivatives (from 1^{st} to 3^{er}) this inconvenient is overcome by using a practical high-gain observer (HGO), which works, as known, like an approximate differentiatior (see [Parlitz, 1995]). The HGO is not based on the Luenberger observers, since it does not require an accurate model.

The remainder of this paper is organized as follows. Section 2 is devoted to studying some important algebraic properties of the Rössler's system. In Section 3, we introduce a simple HGO for estimating the time output derivatives; next, we develop two identification procedures based on the previously introduced algebraic properties. In the observer design and in the two identification schemes, we present computer simulation results depicting their performance. Section 4 is dedicated to giving the conclusions. Finally, in the Appendix we give a brief survey of GA and also present proofs of Propositions 1 and 2.

2 Rossler's System Properties

We consider the popular nonlinear Rössler's System (RS) , which is described by

$$\begin{aligned} \dot{x} &= -(y+z) \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x-c) \end{aligned} \tag{1}$$

It is well known that in a large neighborhood of $\{a = b = 0.2, c = 5\}$ this system presents a chaotic behavior and is considered for exhibiting the simplest possible strange attractor ([Strogatz, 1994]). Originally, the RS, which is credited to Otto Rossler, arose from work in chemical kinetics ([Rössler, 1976]).

The fundamentals of this work are based on the algebraic properties of observability and identifiability that the RS satisfies (see [Martinez & Diop, 2004] and [Martinez & Mendoza, 2003]). The definitions of these are as follow.

Definition 1: *Let us consider an undetermined system of ordinary differential equations*

$$G(t, X, \dot{X}, P) = 0, \tag{2}$$

where $x^T = \{x_1, \ldots x_n\} \in R^n$ *is a state vector and* $P^T = \{p_1, \ldots, p_l\} \in R^l$ *is a constant parameter vector. Suppose that there exists a smooth, local and one to one correspondence between solution* $X(t)$ *of system (2) and an arbitrary function* $y(t) = h(t, X(t)) \in R$ *, then, state* x_i *is said to be algebraically observable with respect to* $y(t)$ *, if it satisfies the following algebraic relationship.*

$$x_i = \frac{f_i\left(y, \ldots, y^{(m)}, P\right)}{g_i\left(y, \ldots, y(s), P\right)}; m \leq s \quad \text{with coefficients in } R \tag{3}$$

where f_i, g_i *and* h *are smooth maps,* $y(k)$ *is the* k^{th} *derivative of* y *and* l, m *are integers. Variable* y *is the output. If* x_i *is observable for every* $i = 1, \ldots, n$*, then we say that the system is completely observable*

Definition 2: *Consider again system (2) under the same conditions of Definition 1. If we can find a smooth map* $W : R^J \to R^l$ *such that*

$$0 = W\left(y, \dot{y}, ..y(j), P\right) \tag{4}$$

then, the parameter vector P *is said to be algebraically identifiable with respect to the selected output*

Next, we verify that RS satisfies the previous definitions when the output is selected as y. Evidently, this system is found to be algebraically observable with respect to the defined output. To see it, variables x and z can be expressed in the following

$$x = \dot{y} - ay \quad ; \quad z = -\ddot{y} + a\dot{y} - y \tag{5}$$

From $\dot{z}$ we can obtain $y^{(3)}$

$$y^{(3)} = -b + \left(\ddot{y} - a\dot{y} + y\right)\left(\dot{y} - ay - c\right) + a\ddot{y} - \dot{y} \tag{6}$$

Therefore, system (1) is identifiable with respect to the output y because the last differential parameterization can be rewritten as,

$$0 = W\left(y, \dot{y}, \ddot{y}, y^{(3)}, P\right); \text{ with } P = [a, b, c] \tag{7}$$

3 States Estimation and Parameters Identification

Since RS is observable and identifiable, then a HGO is proposed to estimate the time derivatives, from the 1st to the 3rd, of the output. Moreover, it is possible to implement an identification scheme to recover the unknown vector P see [Martínez & Diop, 2004]). So, this section is devoted to solving both problems. Firstly, a HGO is proposed to tackle the time derivatives estimation problem. Then, using the observer's estimation of the derivatives, an identification process can be carried out. Finally, we suggest two alternatives for identifying vector P *i)* a traditionally least-square method and *ii)* an algebraic on-line approach.

Before establishing the estimation and identification problems, the necessary assumptions are presented.

A1 *The states of the nonlinear system (1) oscillate around zero.* **A2** *The set of variables* $V_y = \{y(t_k),..., y^{(3)}(t_k)\}$ *are available* $t_k \in \Im = (t_1, t_2, t_N)$, *with a sampling time* τ *selected such that*

$$\tau = t_{j+1} - t_j;\; j = \{1, 2, ..., n-1.\} \tag{8}$$

Notice that A2 will be relaxed by means of an HGO, that is, we estimate with high accuracy the set V_y. Finally, we mention that although the RS identification has been considered by other authors such as [Baker *et al.*, 1996] and [Lainscsek & Gorodnitsky, 1996], they did not use the differential algebraic approach as we did.

3.1 A simple HGO

Based on the previous worksof (see [Dabroom & Khalil, 1999]) and, Bonilla we propose the following HGO.

Let us define vector $Y^T [y, \dot{y}, \ddot{y}, y^{(3)}]$ and let us propose the following filter given by:

$$\dot{\hat{Y}} = A\hat{Y} + HC(Y - \hat{Y}) \tag{9}$$

Where, A is the well-known Brunovsky form

$$H^T = \begin{bmatrix} \frac{\alpha_1}{\varepsilon} & \frac{\alpha_2}{\varepsilon^2} & \frac{\alpha_2}{\varepsilon^3} & \frac{\alpha_2}{\varepsilon^4} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

ε is a small positive parameter and the positive constants α_i are selected such that the polynomial defined as:

$$p(s) = s^4 + \alpha_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \alpha_4, \tag{11}$$

is Hurwitz (see [Dabroom & Khalil, 1999] for more details).

The following proposition allows us to compute the error $\xi = Y - \hat{Y}$.

Proposition 1: *Consider the system (1) under assumptions A1. Then, the HGO proposed in (9) is able to recover* Y *with bounded error*

$$\|\xi\| \leq \beta n\varepsilon / \lambda^{*} \tag{12}$$

Where λ^{*} *is given by*

$$\lambda^{*} = \min\{\mathrm{Re}[roots(p(s))]\}, \tag{13}$$

β *is a positive constant which depends on the initial conditions* $\xi(0)$. *and* $n = \max_{t\in[0,T]}|y^{(3)}(t)|$. **Proof:** The proof is in the Appendix.

Notice that constants ε and λ^{*} are design parameters which can be chosen in order to minimize the error of observation.

3.1.1 *Numerical Simulations* The efficiency of the HGO had been tested by computer simulations. The experiments were implemented by using the 4^{th}-order Runge-Kutta algorithm. The computation was performed with a precision of 8decimal digit numbers, from $t = 0$ second to $t = 10$ seconds. To obtain a good performance, the step size in the numerical method was set to 0.0005. The RS parameter values were set as

$a = 0.25$, $b = 0.2$ and $c = 4.2$, and the initial conditions were set as $x(0) = -1$, $y(0) = 1$ and $z(0) = 0$. The polynomial was chosen to be $p(s) = (s^2 + 2\zeta\omega_n s + \omega_n{}^2)^3$, with $\zeta = 0.0707$ and $\omega_n = 0.9$ The gain of the HGO was selected as $\varepsilon = 0.005$.

Figure 1 shows the error evolution of each output's time derivatives. As can be seen a very good estimation of $y^{(k)}, k = \{1,2,3\}$ is obtained.

3.2 Parameters Identification Based on Least-Squares

Under assumptions A1 and A2 a common quadratic function for estimating vector P from the differential relation (6) is presented as:

$$S(N) = \min_{p\in R^3} \sum_{0}^{k=N} \left(\tilde{y}^{(3)}(t_k, p) - \hat{y}^{(3)}(t_k)\right)^2 \text{ ; with } t_k \in \Im \tag{14}$$

where, symbol "$\tilde{y}^{(3)}(t_k, p)$ denotes the parametric estimator of the 3^{rd} time derivative of y given by

$$\tilde{y}^{(3)}(t_k, p) = -p_1 + (\ddot{y}(t) - p_0\dot{y}(t) + y(t))(\dot{y}(t) - p_0 y(t) - p_2) + p_0\ddot{y}(t) - \dot{y}(t) \tag{15}$$

with $p = [p_0, p_1, p_2] \in R^3$.

Notice that if we try to compute the critical points of relation (14), we need to solve three highly difficult nonlinear parametric equations with respect to parameters $\{a, b, c\}$. That is, obtaining analytically Rössler's parameter values is not possible if solely output y is available. Nevertheless, it is feasible to obtain an algebraic expression for parameters a, b and c as long as the Rössler's states x, y and z can be measured [G.L. Baker *et. al.* ,1996). To overcome these difficulties, we instead use a common GA to compute vector p which minimizes expression (). This method is *ad-hoc*, because, we avoid the necessity of computing derivatives with respect to parameters and of finding the roots of the nonlinear parametric function, and we avoid the possibility of falling into a local minimum (see [Golberg, 1989], [Mitchel, 1998] and [Back *et al.*, 2000]). Finally, we recommend reading a brief description of the GA given in the Appendix in order to understand and to interpret the following numerical simulations.

3.2.1 GA Numerical Simulations To evaluate the efficiency of the proposed identifier method based on GA, a second computer simulation was carried out Basically, we estimate numerically vector p such that expression (14) is minimized by means of a GA . The sampling time was selected $\tau = 0.1$, the number of samples was set to $N = 25$, the cost was selected $\alpha = 10^{-9}$ The samples were taken in the time interval from 6.5 to 9 seconds. Components of vector $p = [p_1, p_2, p_3]$ were searched for in a region centred on $\bar{p} = [0,0,0]$ with radius 7.

Figure 2 shows the process of error minimization. It can be seen that the error tends to zero when the generation number increases. Therefore, the actual and estimated parameters are very close, as shown in Table 1.

Generation	A	B	c
1	0.1789	0.0221	2.6051
3	0.1952	0.3112	4.3115
10	0.2108	0.1624	3.989
32	0.2389	0.2005	4.1167
100	0.2498	0.1999	4.2010

(16)

Table 1.Best individual of some generations

3.3 Identification by solving algebraic linear equations

In this section we relax the identification problem a little in order to obtain an easier solution. Assuming that parameter a is known and the set V_y is available, it is possible to obtain a straightforward solution based on simple linear algebra.

Consider again the differential parameterization (6). This produces, after further time evaluation, the following system of linear equation for the missing parameters, b and c.

$$\begin{bmatrix} -1 & -\omega_1(t_i) \\ -1 & -\omega_1(t_i+\tau) \end{bmatrix}\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} \omega_2(t_i) \\ \omega_2(t_i+\tau) \end{bmatrix} \tag{17}$$

where $\omega_1()$ and $\omega_2()$ are defined as:

$$\begin{aligned} \omega_1(s) &= \ddot{y}(s) - a\dot{y}(s) + y(s) \\ \omega_2 &= y^{(3)}(s) + \dot{y}(s) - a\ddot{y} - \omega_1(s)(\dot{y}(s) - ay(s)) \end{aligned} \tag{18}$$

The linear equation (17) allows us to recover the unknown parameters, b and c, after some time $t_i > 0$, that is to say, at $t = t_i + \tau$, $\tau > 0$, for which the matrix in () is invertible. As we mention in the following proposition:

Proposition 2: *Under assumptions A1 the matrix (17) becomes invertible for almost* $t_i > 0$ *and* $\tau > 0$. **Proof** (Refer to Appendix).

Another possibility to estimate the missing parameters can be done assuming that 4th derivative is available. Now, computing the time derivative of equation (6), this yields:

$$y^{(4)} - a\dddot{y} + \ddot{y} - (\ddot{y} - a\dot{y} + y)(\ddot{y} - a\dot{y}) - (\dddot{y} - a\ddot{y} + \dot{y})(\dot{y} - ay - c) = 0 \tag{19}$$

Evidently, from equations (6) and (19) we can obtain directly the parameters b and c, respectively.

3.3.1 Numerical Simulations We illustrate the effectiveness of the previously described identification method by using inverse matrix. The initial conditions and the physical parameters were taken as indicated in the previous experiment, except for an abrupt change in the Rössler's parameter values b and c, from 0.2 to 0.35 and 4.2 to 3.5 when $t \geq 5$, respectively.

Figure 3 shows the estimation of parameters b and c based on inverse matrix. As a result, the identification process is quite robust with respect to abrupt parameter variations.

4 Conclusions

The differential algebraic approach allowed us to solve the identification problem for the well-known Rössler's attractor. In this instance, we exploited the algebraic properties of observability and identifiability that Rössler's model fulfils, with respect to a very particular available state which is the $y-$ variable. Therefore, we could obtain a differential parameterization of the output and its time derivatives. Based on these facts, an HGO was used for estimating the output's time derivatives, and then, two identification approaches were designed based on the previously parameterization.

The identification approaches were tested by means of numerical experiments. In the first one, a traditionally least-square method was used and solved by a suitable GA, as shown in Table 1; in the second one, the missing parameters were computed in a straightforward way as an inverse matrix. The performance of the inverse matrix was validated in the presence of an abrupt variation in the missing parameters, as shown in figures 2 and 3.

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5 Appendix

A Brief Look at GA (we recommend reading [Bender & Orzag, 1999])

We introduce a traditional GA to minimize expression (14):

(i) Individuals in the GA are vectors (in R^3) of the form

$$q_i = [q_{0,i}, q_{1,i}, q_{2,i}] \quad (20)$$

It is understood that the GA is a real-coded one (as opposed to a binary-coded one).

(ii) Every population consists of 500 individuals.

(iii) The best individual, q_1 (evidently ranked 1st), in generation P_j is passed on to generation P_{j+1}, with no change.

(iv) Several steps are involved in the creation of generation P_{j+1}. They are as follows: *i*) selection; *ii*) crossover; *iii*) mutation.

i) The upper half of P_j passes on to population P^{*}_{j+1} while the lower half is discarded. Note that P^{*}_{j+1} holds just the best250 individuals in P_j.

ii) To accomplish crossover couples, parents are generated as follows: each individual in P^{*}_{j+1} is assigned a probability which is calculated linearly according to its ranking. Selection of individuals is made by generating random numbers in $[0,1]$(say α_i) and comparing them to the accumulated probability , $Ap(q_i)$, of each individual. Individual q_i is selected to be part of P^{*}_{j+1} when $\alpha_i \leq Ap(q_i)$. In a first round a member of each couple is selected while in a second round the other member is selected. The crossover algorithm used in this GA is a slight modification of the *flat crossover (or arithmetic crossover)* operator. An "offspring" $h = [h_0, h_1, h_2,]$is generated as

$$h_i = \beta q_{i,1} + (1 + \beta) q_{i,2}. \quad (21)$$

from "parents"

$$q1 = [g_{0,1}, q_{1,1}, q_{2,1}] \ ; q2 = [g_{0,2}, q_{1,2}, q_{2,2}] \tag{22}$$

where q_1 is a better individual than q_2 (i.e q_1 makes the error function smaller than q_2 does) and β is a random number chosen uniformly from the interval [0.5, 1]. This interval is used in order to weigh as more "influential" the information carried by the best of the parents. This process is repeated until a set P''_{j+1} with 250 ""offspring" is completed. A new population is then created: $P'''_{j+1} = P'_{j+1} \cup P''_{j+1}$.

(iii) The mutation algorithm consists of randomly changing a component of 15% of the individuals of P''_{j+1}. Changes are made within the vicinities specified below. This is the final step in creating generation P_{j+l}

(v) The *cost* of each individual was computed via $S(N,\tau)$ where N is the number of samples and τ is a sampling time. The algorithm stops when the best individual tags a "*cost*" named α , where α is fixed as small as needed.

(vi) Components of vector $q = [g_0, q_1, q_2]$ were searched in a previously defined "box"; this means.

$$q_i - q_i =\leq \varepsilon_i . \ ; i = \{1,2,3\} \tag{23}$$

where q_i is the selected centre and ε_i is the radius which can be chosen as large as needed.

Proof of Proposition 1:

Evidently, vector Y can be written as:

$$\dot{Y} = AY + \delta_y . \ ; \tag{24}$$

with $\delta^T{}_y = [0,0,0, y^{(4)}]$ Subtracting (24) from (9), we obtain the following differential equation of the error:

$$\dot{\xi} = [A - HC]\xi + \delta_y . \tag{25}$$

Notice that the characteristic polynomial of $A = A - HC$ is given by $p(s\ \varepsilon)$, wich is also Hurtwitz. That is, the proposed H assigns the eigenvalues of A at $1\ \varepsilon$ times the roots of $p(s)$ (11). Hence, the error ε satisfies

$$\varepsilon(t) = e^{A(t-t_0)}\left(\varepsilon(0) + \int_{t_0}^{t} e^{A(t_0-s)}\delta_y(s)ds\right). \tag{26}$$

Since A is exponentially stable and the signal $y^{(3)}$ is bounded, we also have the following inequality:

$$\xi \leq \beta e^{-\lambda^* t}\xi + \beta\eta\varepsilon\left(1 - e^{-\lambda^* t}\right)\lambda^* \to \beta\eta\varepsilon\ \lambda^*. \tag{27}$$

where the positives constants β, λ^*, η are previously defined in proposition 1.

Proof of Proposition 2:

Let $\Delta(t_i, \tau)$ be the determinate of matrix (17) which is given by $\Delta(t_i, \tau) = -\omega_1(t_i, \tau) + \omega_1(t_i)$. From the definition of variable $\omega_1()$ given in (18) and taking into account system (17), we obtain after some simple algebra, the following:

$$\Delta(t_i, \tau) = -z(t_i + \tau) + z(t_i). \tag{28}$$

Since the variable $z(t)$ oscillates around zero, we must have $-z(t_i + \tau) + z(t_i) \neq 0$ for almost $t_i > 0$ and $\tau > 0$. As the matter of fact $-z(t_i + \tau) + z(t_i) = 0$ only in a finite set of time. $\tau = \{\tau_1, \tau_2, \ldots\ldots, \tau_n\}$. Therefore, matrix (17) is invertible for almost $t_i > 0$ and $\tau > 0$.

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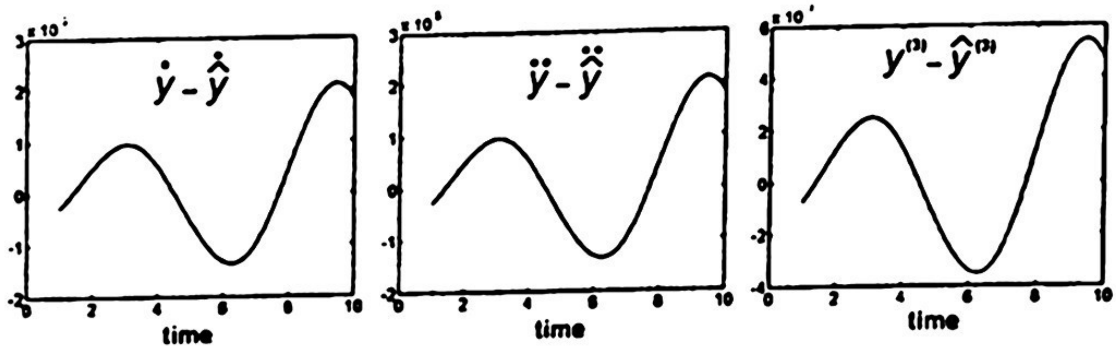

Fig. 1. The error evolution of each output's time derivatives.

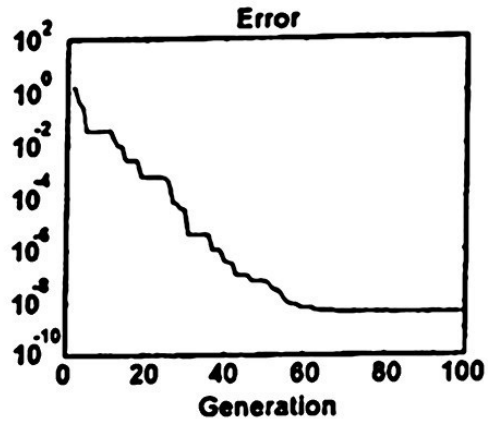

Fig. 2. Process of error minimization.

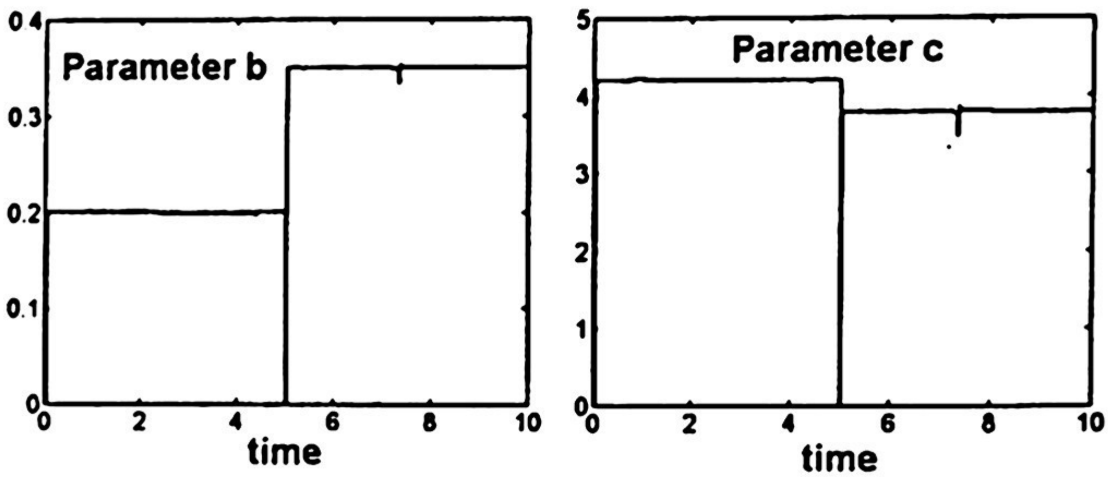

Fig. 3. The estimation of parameters b and c.